Experiment: VAK (12/09/2002)

Contents:

1. Introduction

This experiment provides an insight into today's vacuum technology, and can also be seen as an opportunity to recollect the characteristics of an ideal gas. The pumping speed of a rotary-slide valve vacuum pump in different experimental situations will be investigated. Doing the necessary pressure measurements, we will – among other things – get to know a Pirani manometer, which allows to determine the pressure indirectly by measuring the thermal conductivity of the gas.

2. Experimental Tasks

2.1 Calibration of the Pirani manometer

The Pirani manometer used in this experiment is – in principle – a Wheatstone bridge consisting of three 120Ω resistors and – as fourth resistor – a resistance wire (made of tungsten), which runs through the gas volume. The real assembly and the circuit are shown in fig. 2.1.1.

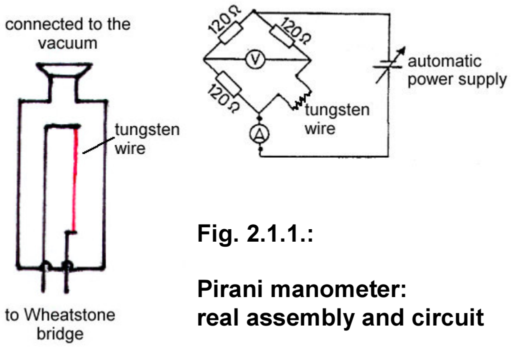


Fig. 2.1.1.:

Pirani manometer: real assembly and circuit

At small pressures (around 10^{-3} – 1 hPa), the thermal conductivity of a gas (and therefore the heat flux from the resistance wire into the gas, when a current flows through the bridge) depends on the density of the gas. An automatically controlled power supply regulates the current through the bridge (and therefore the heat dissipation) in such a way that the bridge is always adjusted[1]; this is the case if the temperature of the wire is constant and the resistance of the wire equals 120Ω. Constant temperature, however, means that the influx of electric energy into the resistance wire equals the heat flux from the wire into the gas; therefore, in this case, the current as a measure of electric power can give us information on the thermal conductivity of the gas and thus the pressure. To measure the current, we included an ammeter in the circuit like shown in fig. 2.1.1.

The correlation between the current through the tungsten wire and the pressure is unknown at the beginning of the experiment; this means we have to obtain the (bijective) function $p \Leftrightarrow I_{Pirani}$ using a reference manometer first. In this case, this is a combined assembly which contains a u-tube manometer and a McLeod manometer (functional principle see instructions). Both gauges contain calibrated scales, where the pressure can be read off. The McLeod gauge can be used at pressures up to 1.5hPa; the u-tube manometer works at pressures above this value. The McLeod manometer is very exact, if one obeys the rules; that means that the measuring process may not be started before the pressure is stable and the same throughout the evacuated volume (see instructions). When we used the u-tube manometer, in contrast, it showed up that it has two pressure scales which show quite different values. We thus noted down mean values; but those may also not be very precise, as the scale cannot be read off as exactly as the scale of the McLeod.

The following set-up was used for this part of the experiment:

When the pump is on, you can regulate the resulting pressure in the system by opening or closing the dosing valve; pressures above 10hPa however cannot be reached if the pump is on, even if the dosing valve is totally open. The measurements at the higher pressures were therefore made with the pump switched off; the dosing valve has to be closed for every measurement in this case and must be opened between the measurements so that enough air can get into the system and a higher pressure is reached.

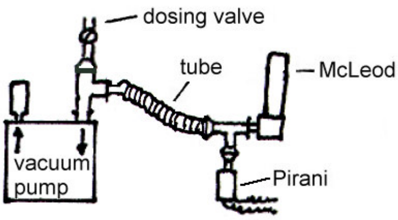


Fig. 2.1.2: arrangement for the calibration process

After these explanations we finally show the values we obtained. We have prepared two tables, one showing the correlation $p \Leftrightarrow I_{Pirani}$ and one showing the correlation $p \Leftrightarrow W_{Pirani,resistance\ wire}$, where $W_{Pirani,resistance\ wire}$ equals ¼ * 120Ω * I_{Pirani}^2 according to the instructions[2].

[1] A wheatstone bridge is called „adjusted“ if there is no current through the diagonal connection, see experiment BRÜ.
[2] The meaning of the factor ¼ becomes clear, if one looks at the circuit: The total resistance of the Wheatstone bridge equals 120Ω (!) here and one fourth of the total electric power ($R * I^2$) is dissipated at every resistor built into the bridge, and therefore also at the resistance wire.

p [mbar]	I [mA]	lg(p/mbar)	lg(I/mA)	ΔI [mA]	Δp [mbar]
1,00E-04	2,7	-4,00	0,43	-	-
5,00E-03	3,2	-2,30	0,50	0,7	0,002
0,020	3,9	-1,70	0,59	0,7	0,005
0,031	4,4	-1,51	0,64	0,7	0,005
0,040	4,9	-1,40	0,69	0,7	0,005
0,070	5,7	-1,15	0,76	0,7	0,005
0,080	6,0	-1,10	0,78	0,7	0,005
0,10	6,6	-1,00	0,82	0,7	0,02
0,15	7,8	-0,82	0,89	0,7	0,02
0,23	9,6	-0,64	0,98	0,7	0,02
0,32	11,2	-0,49	1,05	0,7	0,02
0,55	14	-0,26	1,15	3,3	0,02
0,92	17	-0,04	1,23	3,3	0,02
1,06	19	0,03	1,28	3,3	0,04
1,54	22	0,19	1,35	3,3	0,04
8	33	0,90	1,52	3,3	4
16	37	1,19	1,57	3,3	4
28	43	1,44	1,63	3,3	4
37	45	1,57	1,65	3,3	4
59	48	1,77	1,68	3,3	4
81	50	1,91	1,69	3,3	4
101	51	2,00	1,70	3,3	4

p [mbar]	P [mW]	ΔP [mW] (err. propag.)	Δp [mbar]
1,00E-04	0,2	-	-
5,00E-03	0,3	0,13	0,002
0,020	0,5	0,16	0,005
0,031	0,6	0,18	0,005
0,040	0,7	0,21	0,005
0,070	1,0	0,24	0,005
0,080	1,1	0,25	0,005
0,10	1,3	0,28	0,02
0,15	1,8	0,33	0,02
0,23	2,8	0,40	0,02
0,32	3,8	0,47	0,02
0,55	6	2,79	0,02
0,92	9	3,35	0,02
1,06	11	3,76	0,04
1,54	15	4,40	0,04
8	33	6,53	4
16	41	7,35	4
28	54	8,42	4
37	61	8,91	4
59	69	9,50	4
81	74	9,80	4
101	77	10,00	4

Before we show the diagrams, we want to say some words about the tables: The first thing which stands out in the first table is, that we have also noted values for lg(I) and lg(p). The sense of these values will be explained in a short wile, as well as the meaning of the different colours. But now, there's still something mysterious about the tables:

How did we obtain the measurement errors for the different values?

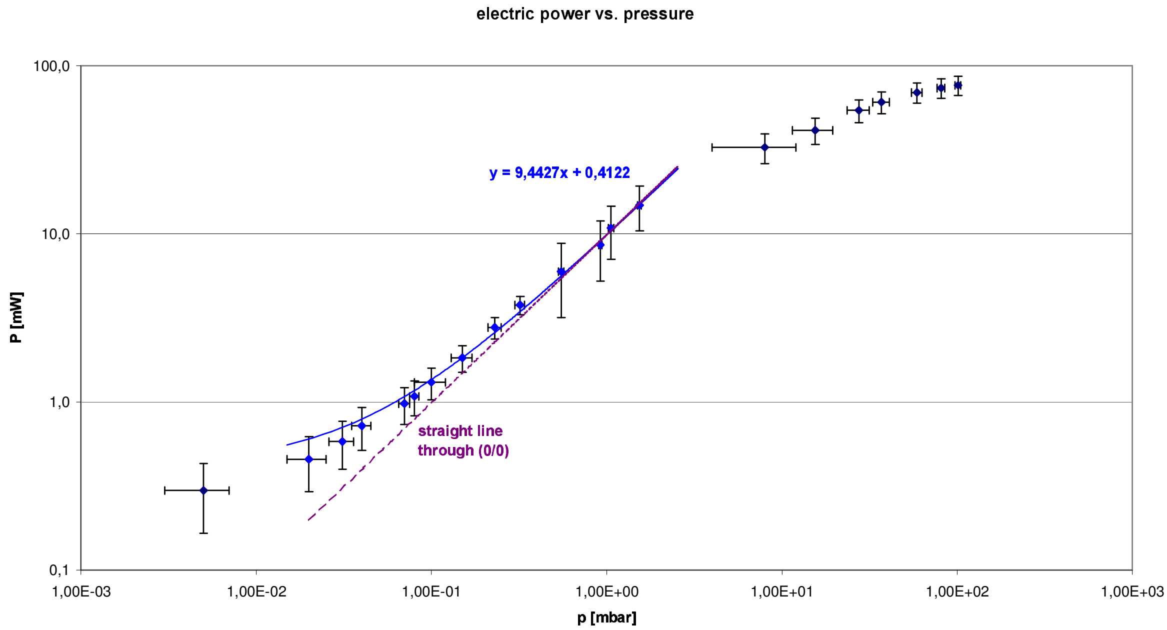
The uncertainty for p is an estimated error, which emerges from reading out the scale at the manometer; the error value for I is obtained through linear summation of the statistic error caused by inexactnesses reading out the scale and the systematic error caused by the ampere meter[3].

The error for $P = ¼\ R\ I^2$ is a little bit harder to obtain: We have to do an error propagation (neglecting the error for R according to the instructions) and get:

$$\Delta P = (\partial P/\partial I) * \Delta I = ½\ R\ I * \Delta I = ½ * 120\Omega * I * \Delta I$$

[3] The ammeter was marked as „class 5“ device, that means that the possible systematic error caused by erroneous calibration equals 5% of the highest value gaugeable in the active metering range.

Now, we can finally show the first diagram we had to make; that is the one for P(p). The red values in the tables above are excluded from any further discussion and calculations and diagrams because of the very great uncertainty of the McLeod manometer scale at 0.0001 mbar.



After adding the values and the corresponding error bars into the doubly logarithmic coordinate system, we had to find out in which region the function $P(p)$ is a linear one. We have to remember the appearance of different function types on doubly logarithmic paper:

For every function of the type $f(x) = a*x^b$, the relation $\lg(f(x)) = \lg(a) + b*\lg(x)$ is valid. Therefore, any functions of this type (and not only linear functions) will appear as straight lines on doubly logarithmic paper.

Linear functions can be of the type $f(x) = a*x$ (in this case we'll see a straight line with a slope of 1 on logarithmic paper) or of the type $f(x) = a*x + b$; in the latter case we will see a line like the light blue one in the diagram above (b has to be nonnegative for logarithmic scaling if the function is to be displayed for x -> 0).

According to the instructions, equation no. (9), $P(p)$ can only be a function of the type $P(p) = a*p + b$ (where b represents unwanted effects causing a heat flux out of the resistance wire even if the pressure equals zero, see below) for small pressures. We have to search for a range, in which the function $P(p)$ is truly linear. That means, we not only have to take a look if a straight line can be fitted to the measured values, but we must also take care that its slope on the logarithmic paper equals 1, because $P(p)$ is <u>not</u> of the type $a*p^q$, $q \neq 1$. Therefore, we try to generate a linear regression curve $P(p) = a*p + b$ with $b = 0$ as a first step. The first dark blue value pair in the table is excluded from the regression because its relative measurement error is quite high; values with pressures larger than 1.54 mbar are also excluded because flattening curves on a doubly logarithmic paper can not represent linear functions[4]. The result is the violet curve, but obviously it doesn't represent the real correlation $P \Leftrightarrow p$ very well. P is almost proportional to p only for 0.5mbar < p < 2mbar.

[4] That means that for values around 8mbar and above, eq. (9) obviously isn't valid any more.

If we take a curve of the type

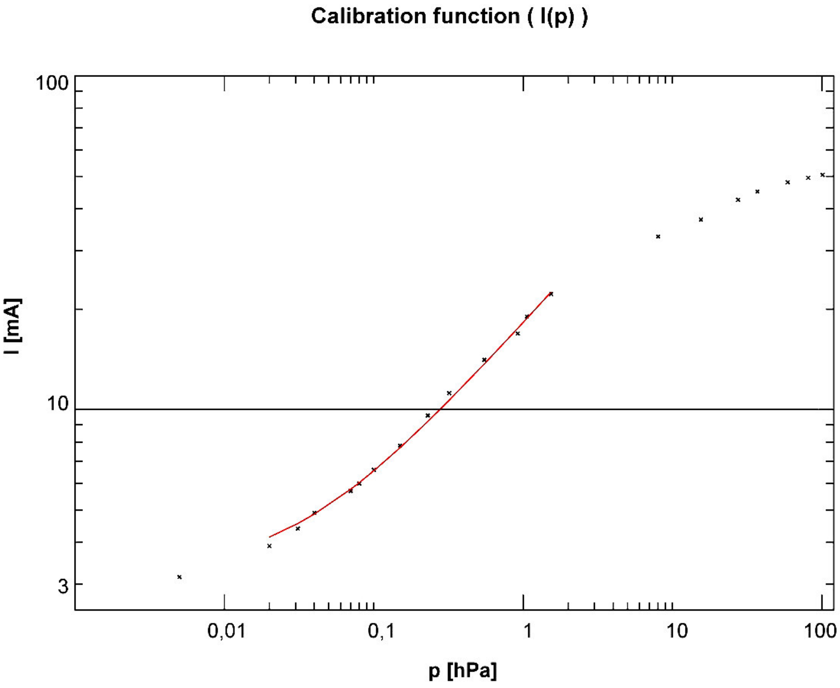
$$\text{(I)}: P(p) = a*p + b$$

with $b \neq 0$ however, we get a much better regression curve – this is the blue line in the diagram. We can conclude that such a linear correlation $P(p)$ exists for 0.015mbar < p < 2mbar.

But why isn't P simply proportional to p for small pressures, as equation (9) says? Why is b greater than zero? The explanation is quite simple: The heat produced in the resistance wire is not only flowing into the gas, but a part of it can also - for example - flow off through the package of the Pirani manometer or be emitted as heat radiation. Therefore, the heat flux out of the resistance wire equals the heat flux into the gas plus a constant, which is represented through b. That explains, why the curve for $P(p)$ flattens at its left end.

After this analysis, we finally want to get a calibration relation, that means, we want to find a method which gives us p if we know I.

We start off with a doubly logarithmic diagram again, this time for $I(p)$; the values were taken from the table shown on page 3:



After a short look onto the diagram, we decided to use the following system for determining the corresponding pressure to a given current value when analysing the parts 2.2 and 2.3 of the experiment:

- If the current is greater than 3.55mA and smaller than 25mA (this criteria is approximately equivalent to 0.015mbar < p < 2mbar)…

  … $P \equiv \frac{1}{4} R I^2$ can be expressed as $P(p) = a*p + b$. That means that I can be expressed as:

$$\text{(II)}: (I/\text{mA}) = (c_1 * (p/\text{mbar}) + c_2)^{0.5} \text{ with two constants } c_1 \text{ and } c_2 \in \text{R}.$$

We used the program xmgrace to fit a curve of this type for $I(p)$ to our values; the result is the red curve shown in the diagram. The program directly gives us the computed values for c_1 and c_2:

$$c_1 = 322.098;\ c_2 = 10.7578$$

From (II) and this information, we can calculate $p(I)$:

$$p/\text{mbar} = ((I/\text{mA})^2 - 10.7578) / 322.098 \text{ (for } 3.55\text{mA} < I < 25\text{mA)}$$

- What if $I < 3.55$mA or $I > 25$mA?

  In this cases we decided to calculate the pressure by linear interpolation in a doubly-logarithmic diagram. That means, at first we imagine a doubly-logarithmic diagram for $p(I)$ (!) with the measured values. If we now want to get the corresponding pressure for a current value of 25mA, for example, we look for the two measured values for a current directly above and directly below 25mA. Then, we calculate a straight line (in the doubly logarithmic diagram) running directly through the two measured points and use the corresponding equation

$$\text{(III)}: (\lg(p) = \lg(a) + b * \lg(I))$$

  for calculating $p(I)$. If the value of I is below the lowest measured value, we take the two measured points with lowest I values for calculating the straight line; an analogous method is used for current values above the highest measured value.

Because all of this is very complicated, we implemented this algorithm in a macro which can be executed by our spreadsheet program. The macro makes use of the lg(p) and lg(I) values shown in the first table; the source code is attached and explained in appendix A.

2.2 Pumping speed

The first "real" experimental task was to measure the pumping speed S of the pump. Let us have a look at the experimental set-up:

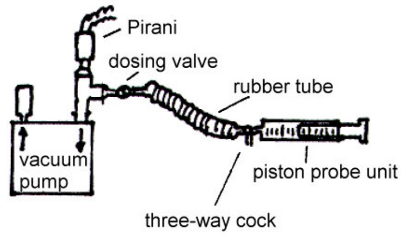


Fig. 2.2.1: arrangement for the pumping speed measurement

The first step is to generate a constant vacuum in the system, while the three-way-cock is set to allow the flow of air from outside the system into the rubber tube. The dosing valve is adjusted so that the current through the Pirani manometer is constant and the corresponding pressure is about 0.5 hPa.

In our case, the exact measured current was:

$$I_{Pirani} = 13.1\text{mA} \ \pm 3.3\text{mA}$$

The error of 3,6mA is calculated through linear addition of the systematic ammeter error (3,0mA = 5% * 60mA) and the noticed variation of the current (± 0,3mA) during the experiment.

That means, that the pressure inside the system was:

$$p = 0.50\text{hPa}$$

If we calculate the pressure values for I_{Pirani} = 16.4mA respectively 12.8mA with the help of our macro, we see that this value of p is affected by an error of +0.30hPa / -0.024hPa, that is equivalent to a relative error of +60% / -5%. We consider the error of the Pirani manometer calibration and roughly estimate the total error for p to:

$$+70\% \, / \, -15\%$$

After having measured the constant pressure, the three-way cock is switched so that it only connects the rubber tube with the piston probe unit. The unit, filled with 100ml of air at about 1000hPa gets evacuated; on a scale printed onto its glass, one can watch the remaining amount of air. We noted down the time which the pump needed for reducing the volume from 100ml to 90ml, from 90ml to 10ml and so on; this series of measurements was recorded three times. The measured values, as well as an average value and its standard deviation (calculated with the help of a spreadsheet program) are shown in this table:

Volume reduction	ΔT [s]		
from -> to [ml]	series 1	series 2	series 3
100 -> 90	25,0	25,0	24,0
90 -> 80	23,0	21,0	21,5
80 -> 70	22,0	22,5	20,5
70 -> 60	21,5	23,0	23,0
60 -> 50	22,0	24,5	24,0
50 -> 40	22,0	23,5	23,0
40 -> 30	23,5	23,0	24,0
30 -> 20	21,5	24,5	22,5
20 -> 10	22,5	23,0	23,0
10 -> 0	24,0	20,0	21,0
ΔT average [s]:	**23**		
std. deviation [s]:	**1,3**		

With the help of our results, we can now calculate S, which is according to the instructions:

$$S = \frac{d\left(p_{air} \cdot V_{air}\right)}{dt} \cdot \frac{1}{p_0} \overset{p_{air}=const.}{=} p_{air} \cdot \frac{dV}{dt} \cdot \frac{1}{p_0} = \frac{1000\text{hPa}}{0{,}5\text{hPa}} \cdot \frac{10\text{ml}}{\overline{\Delta \text{T}}} = \frac{1000\text{hPa}}{0{,}5\text{hPa}} \cdot \frac{10\text{ml}}{23\text{s}} = 8.7 \cdot 10^2 \, \frac{\text{ml}}{\text{s}} = 3.1 \frac{\text{m}^3}{\text{h}}$$

(In this equation, p_0 is the pressure measured at the pump with the help of the Pirani gauge).

The obtained value is a not as high as the value given by the company (3.7 m³/h). We now calculate the uncertainty for S assuming an average value of ±43% (i.e. ±0.22hPa) as the error of p_0. The error of the average value for ΔT equals the standard deviation given in the table.

$$\Delta S = \left| \frac{\partial S}{\partial \overline{\Delta T}} \cdot \Delta\left(\overline{\Delta T}\right) \right| + \left| \frac{\partial S}{\partial p_0} \cdot \Delta p_0 \right| = \frac{1000\text{hPa} \cdot 10\text{ml}}{0.5\text{hPa} \cdot \left(\overline{\Delta T}\right)^2} \cdot 1.3\text{s} + \frac{1000\text{hPa} \cdot 10\text{ml}}{p_0^2 \cdot 23\text{s}} \cdot 0.22\text{hPa} \overset{\substack{\overline{\Delta T}=23\text{s} \\ p_0=0.5\text{hPa}}}{=} 1.6 \frac{\text{m}^3}{\text{h}}$$

This seems not realistic at first, but taking into consideration the circumstances (calibration, relatively bad ammeter…), one realizes where this error comes from. So, one can say that the measured pumping speed equals the value given by the company within the scope of the uncertainty.

2.3 Effective pumping speed (pumping time)

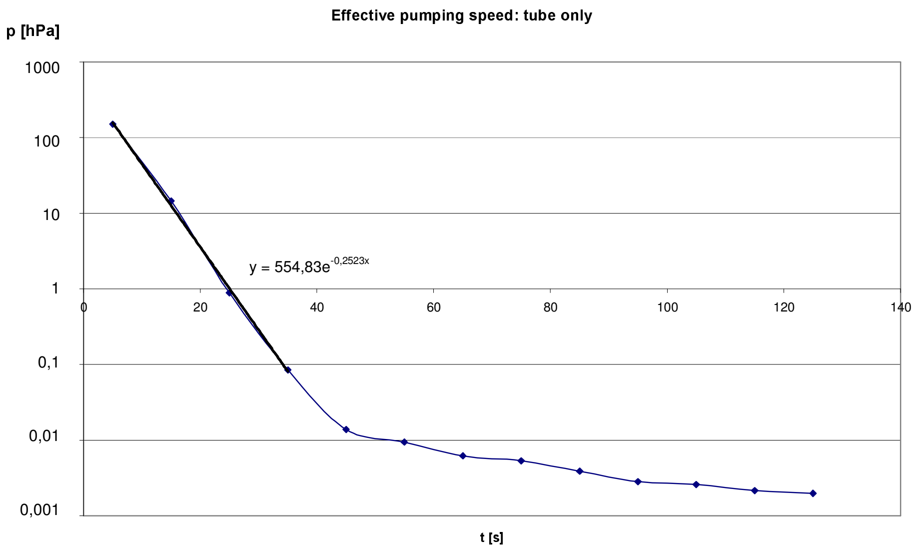
In the third part of the experiment we had to determine the effective pumping speed when evacuating the brass vessel (volume $V = (3.0 \pm 0.1)\ \text{l}$) using different connections between recipient and pump.

- **Tube only**

First, the vessel was connected to the pump by a tube of 25mm diameter. After having started to pump, the Pirani-manometer-current was measured every 10 seconds with a total pumping-time of 2 minutes. This procedure was carried out three times. From the three measurements we calculated a mean value for the current to each point of time. Using these mean values we determined the respective pressures with the macro described earlier. Here are the results:

t [s]	I [mA] meas. 1	meas. 2	meas. 3	mean value [mA]	p [hPa]
5	52,0	52,5	52,5	52,33	149,7
15	36,0	38,5	35,5	36,67	14,50
25	16,5	19,0	16,0	17,17	0,8815
35	6,0	6,5	6,0	6,17	0,0847
45	3,9	3,9	3,9	3,90	0,0138
55	3,50	3,42	3,50	3,47	0,00943
65	3,30	3,22	3,24	3,25	0,00617
75	3,20	3,18	3,16	3,18	0,00532
85	3,04	3,00	3,05	3,03	0,00389
95	2,98	2,80	2,87	2,88	0,00282
105	2,90	2,83	2,81	2,85	0,00259
115	2,80	2,75	2,75	2,77	0,00215
125	2,75	2,72	2,72	2,73	0,00198

From these values we can now plot the pressure as a function of the pumping-time in semi-logarithmic scale:



As described in the instructions the curve should be a straight line, since there is an exponential relation between pressure and time:

$$p(t) = p_0 \cdot \exp\left(-\frac{S_{eff}}{V} \cdot t \right)$$

However, our results show that the curve is linear only at the beginning of the measurements. After about 40s the slope decreases. Therefore we have calculated an exponential regression for the first values only (it looks linear in semi-logarithmic scale, of course). The equation for this regression function can also be taken from the diagram:

$$p(t) = 554.83 \cdot \exp\left(-0.2523 \cdot t / \mathrm{s}\right) \mathrm{hPa}$$

If we compare the last two equations we find the following expression from which it is possible to calculate the effective pumping speed out of the slope:

$$\frac{S_{eff}}{V} = 0.2523 \mathrm{s}^{-1} .$$

Using the vessel's volume V=3.0l=0.003m³ this leads to:

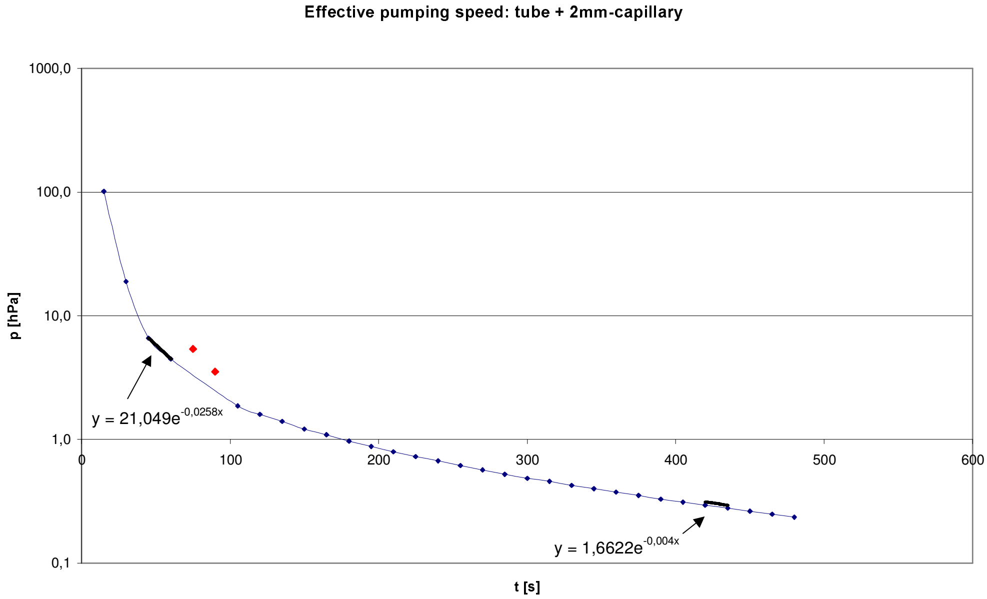
$$S_{eff,tube} = V \cdot 0.2523 \approx 2.72 \frac{\mathrm{m}^3}{\mathrm{h}} .$$

This effective pumping speed is valid for pressures within the validity of the regression function, i.e. approximately for 0.08hPa $< p <$ 150 hPa.

- **Tube + capillary of 2mm diameter**

In another measurement the vessel was connected to the pump by the tube and a capillary of 2mm diameter (connected in series). Again we recorded the Pirani-current in intervals of 15s with a total measuring time of 8 minutes (one measurement only). From the currents the pressures were calculated as before. Our results are:

t [s]	I [mA]	p [hPa]
15	50,5	101,0
30	38,9	18,93
45	31,5	6,594
60	28,7	4,478
75	30,0	5,383
90	27,1	3,528
105	24,71	1,862
120	22,93	1,599
135	21,49	1,400
150	20,06	1,216
165	19,01	1,089
180	17,99	0,971
195	17,15	0,8797
210	16,33	0,7945
225	15,66	0,7280
240	15,04	0,6689
255	14,43	0,6131
270	13,91	0,5673
285	13,4	0,5241
300	12,91	0,4840
315	12,58	0,4579
330	12,15	0,4249
345	11,82	0,4004
360	11,46	0,3743
375	11,14	0,3519
390	10,82	0,3301
405	10,55	0,3122
420	10,27	0,2941
435	10,01	0,2777
450	9,76	0,262
465	9,53	0,249
480	9,30	0,235



The red-coloured values obviously do not fit into the curve and therefore have been left out (we are not quite sure about the reason for these values but we assume a measuring error).

We were now asked to determine the effective pumping speed for two different pressures, 5hPa and 0.3hPa. Since the curve is obviously not linear in semilogarithmic scale (that means the relation between *p* and *t* is not exponential), we decided to fulfil this task as follows: for each pressure we first determined the two points in the diagram lying next to the respective pressure value (but on different sides of it, i.e. one with a slightly higher and another one with a lower pressure). Out of these points we calculated an exponential regression function (which again looks linear in the diagram) for each of the two pressure values, which approximates the curve near the pressure of interest. The respective equations can be taken from the diagram. They contain the slope of the curve (the exponent) at the respective position from which we can get the effective pumping speed like we have done above.

At a pressure of **5hPa** this leads to the following regression function:

$$p(t) = 21.049 \cdot \exp\left(-0.0258 t/\mathrm{s}\right)\mathrm{hPa}$$

Using this, the effective pumping speed is

$$S_{eff}\left(5\mathrm{hPa}\right) = V \cdot 0.0258 \approx 0.28\frac{\mathrm{m}^3}{\mathrm{h}}.$$

With ***p*=0.3hPa** we get the regression function

$$p(t) = 1.6622 \cdot \exp\left(-0.0040 \cdot t/\mathrm{s}\right)\mathrm{hPa}$$

and the effective pumping speed

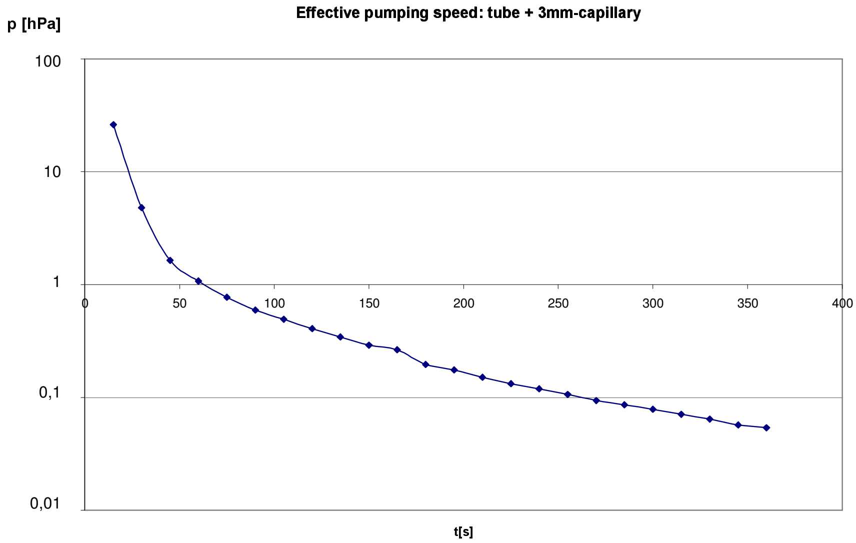
$$S_{eff}\left(0.3\mathrm{hPa}\right) = V \cdot 0.0040 \approx 0.043\frac{\mathrm{m}^3}{\mathrm{h}}.$$

- **Tube + capillary of 3mm diameter**

In the last part of the experiment the capillary was replaced by another one of 3mm diameter. The current was recorded every 15s again with the total measuring time now being 6 minutes (one measurement only). The pressures were calculated as before.

t [s]	I [mA]	p [hPa]
15	42,0	26,2
30	29,2	4,81
45	23,2	1,64
60	18,9	1,08
75	16,1	0,771
90	14,2	0,593
105	13,0	0,491
120	11,9	0,406
135	11,0	0,342
150	10,2	0,290
165	9,8	0,265
180	8,6	0,196
195	8,2	0,175
210	7,7	0,151
225	7,3	0,132
240	7,0	0,119
255	6,7	0,106
270	6,4	0,094
285	6,2	0,086
300	6,0	0,078
315	5,8	0,071
330	5,6	0,064
345	5,4	0,057
360	5,3	0,054

Plotting the pressure as a function of the pumping time leads to the following diagram:



One can see that the curve is rather similar to the one we got with the 2mm-capillary but, of course, the evacuation of the vessel is clearly faster now, i.e. it takes much less time to lower the pressure to a certain value.

We are now going to calculate the theoretical estimates for the conductance of the tube and the 2mm-capillary using the equations given in the manual. The way this can be done depends on whether we have to assume a viscous or a molecular gas flow. This is why we will perform the following calculations with two different pressures, 5hPa (viscous flow) and 0.3hPa (the flow can be taken as molecular).

***p* = 5hPa = 500N/m²:**

In case we assume viscous flow, the conductances can be calculated from equation (18) of the manual:

$$L = \frac{\pi \cdot d^4}{128 \cdot \eta \cdot l} \cdot p$$

Here, η is the viscosity of air ($\eta = 1.82 \cdot 10^{-5}\,\mathrm{kgm^{-1}s^{-1}}$), *p* the pressure, *l* the length of the tube or capillary and *d* its diameter. Using this we get the following results:

	l [cm]	*d* [mm]	*L* [m³/h]
tube	63.0 ± 0.5	25 ± 1	1505 ± 121
capillary	9.5 ± 0.2	2.0 ± 0.1	0.409 ± 0.042

When calculating the errors given in the last column of the table one may use that it is possible to add up the relative errors of each variable, since the right hand side of the equation given above is a product. Therefore the relative error of *L* is

$$\frac{\Delta L}{L} = \sqrt{\left(\frac{\Delta l}{l}\right)^2 + 4\cdot\left(\frac{\Delta d}{d}\right)^2} .$$

Now we are able to calculate the effective pumping speed $S_{eff,theoretical}$ when using tube and 2mm-capillary as connection between pump and vessel. As described in the instructions, we use

$$\frac{1}{S_{eff,theoretical}} = \frac{1}{S} + \frac{1}{L_{tube}} + \frac{1}{L_{capillary}} \quad \Leftrightarrow \quad S_{eff,theoretical} = \frac{1}{\frac{1}{S} + \frac{1}{L_{tube}} + \frac{1}{L_{capillary}}} .$$

Here, S is the pumping speed of the pump: S=3,7m³/h (this is the value given by the company).

From the conductances in the table above we get as **effective pumping speed of tube + 2mm capillary connected in series**:

$$S_{eff,theoretical,2mm}\left(5\text{hPa}\right) \approx \left(0.37 \pm 0.03\right)\frac{\text{m}^3}{\text{h}} .$$

The error was calculated through quadratic error propagation from the errors of L_{tube} and $L_{capillary}$:

$$\Delta S_{eff} = \sqrt{\left(\frac{\partial S_{eff}}{\partial L_{tube}}\cdot \Delta L_{tube}\right)^2 + \left(\frac{\partial S_{eff}}{\partial L_{capillary}}\cdot \Delta L_{capillary}\right)^2} =$$

$$= \sqrt{\frac{\left(\Delta L_{tube}\right)^2}{\left(\frac{1}{S} + \frac{1}{L_{tube}} + \frac{1}{L_{capillary}}\right)^4 \left(L_{tube}\right)^4} + \frac{\left(\Delta L_{capillary}\right)^2}{\left(\frac{1}{S} + \frac{1}{L_{tube}} + \frac{1}{L_{capillary}}\right)^4 \left(L_{capillary}\right)^4}}$$

***p* = 0.3hPa = 30N/m²:**

At this pressure we have to assume molecular flow. The conductance can then be calculated as follows:

$$L = 121\cdot\frac{d^3}{l}\frac{\text{m}^3}{\text{s}} = 3600\cdot 121\cdot\frac{d^3}{l}\frac{\text{m}^3}{\text{h}}$$

The results can be seen from the table. All error calculations are absolutely analogous to the ones for the higher pressure.

	***l* [cm]**	***d* [mm]**	***L* [m³/h]**
tube	63.0 ± 0.5	25 ± 1	10.8 ± 0.8
capillary	9.5 ± 0.2	2.0 ± 0.1	0.037 ± 0.003

With these values we get the effective pumping speed:

$$S_{eff,theoretical,2mm}\left(0.3\text{hPa}\right) \approx \left(0.037 \pm 0.003\right)\frac{\text{m}^3}{\text{h}}$$

From our results it becomes clear that it is very important to dimension the diameters of tubes and pipes adequately when constructing a vacuum set-up. Actually the diameter enters the conductance as third power at least (when assuming molecular flow, with viscous flow even as fourth power) so that it will hardly be possible to reach a sensible effective pumping speed with the tubes and pipes being too thin even if the pumping speed of the pump itself is increased.

Finally we want to compare our theoretical results to the experimental ones:

eff. pumping speed (tube+2mm-cap.)	p=5hPa	p=0.3hPa
theoretical [m³/h]	0.37 ± 0.03	0.037 ± 0.003
experimental [m³/h]	0.28	0.043

Obviously our experimental values do not correspond to the theoretical expectations very well. The reason for this is that there are a lot of possible errors in our measurement that we can hardly calculate. For example, any leak in our system will cause a value lower than the one we expected (as it is with the one for p=5hPa). This might also be the reason why our curves do not remain linear (in semilogarithmic scale) during the whole measurement (as we would have expected from the theory).
It is also quite clear that the regression functions we used to determine our experimental values were calculated from two points only and therefore may not be very precise (unfortunately this was the only way to cope with the changing slope of the curve). Since the ammeter was rather bad, the current through the Pirani-manometer can have a non-negligible error which then falsifies the regression functions (the low number of points from which they are calculated makes it impossible to compensate statistical errors – any systematic errors of the ammeter play a role in addition and could not be eliminated by regression).
Furthermore, all the pressures could only be found by interpolating the calibration function we recorded at the beginning. Any error in this function will, of course, have systematic influences on the pressures from which the pumping speed was calculated. This effect may be very distinct, since the possible errors in the calibration function are probably quite large.
The last two sources of errors (in contrast to the first one) may also explain our experimental value for p=0.3hPa, which, surprisingly, is even higher than the one we would have expected from theory.

2.4 Serial and parallel connection of capillaries

Due to lack of time, we were not able to do this additional work.

3. Questions

3.1 Ideal gas

The concept of the ideal gas is a model to describe the properties of gases in a certain range of pressure and temperature. In this model, the gas atoms or molecules are described as spheres without any volume (i.e. points) that do not interact with each other but through elastic collisions. From this basic idea equations can be derived to describe the properties of such a gas. A very important result of the theory is the ideal gas law

$$p \cdot V = N \cdot k \cdot T$$

(p: pressure, V: volume, N: number of particles, k: Boltzmann constant, T: absolute temperature).
Any gas that obeys this law is called ideal. Real gases, of course, will generally not be ideal, since the atoms or molecules have finite volumes and can exert forces on each other (electrostatic ones mainly). However, the concept of ideal gases can still be used under certain circumstances, as long as the interaction between the particles apart from elastic collisions can be neglected. This will usually be possible at sufficiently low pressures and sufficiently high temperatures, such as with the air in our experiment. Low pressure at relatively high temperature means that the density of gas particles in a given volume is low, so that the atoms or molecules predominantly have long distances from each other. Therefore, their interaction is negligible and the gas approximately shows the properties of an ideal one.

3.2 Thermal conductivity

To understand the thermal conductivity of gases, we first have to get straight in our mind what heat actually is. In fact, temperature is nothing but a measure for the mean kinetic energy of the (here: gas-) particles. The higher the temperature is, the faster move the atoms or molecules (of course, the particles do not all have the same speed but there is a simple statistical connection between the absolute temperature T and the mean kinetic energy : $\overline{E_{kin}} = \frac{3}{2}kT$).
Let us now imagine a vessel filled with gas. If we start to heat up the gas at one end of the vessel, the mean kinetic energy of gas particles at this end will rise as the temperature increases. Other atoms/molecules near the other end of the vessel will not be affected by this change at first. However, the faster particles from the first end will soon pass on energy to other (slower) ones when colliding elastically with them. Therefore, heat energy will finally spread out from the heated end of the vessel into the whole volume, as the energy is passed on as described above. From this we can see that heat can be transported through a gas.
Moreover it is quite clear that not all gases will be equally good conductors of heat under all circumstances (for example the particle density and therefore the pressure could play a role here, cp. question 3). Thus it makes sense to describe this ability of heat transportation quantitatively. The corresponding quantity is called the thermal conductivity of the gas.

3.3 Vacuum flask

If the thermal conductivity was pressure independent at all pressures it would not make sense at all to evacuate the envelope of a vacuum flask. In fact, however, this independence is only true as long as the pressure is not too small. At small pressures the thermal conductivity becomes proportional to the particle number density and therefore (if the temperature remains constant) to the pressure of the gas as described in the manual to the experiment (equations (8) and (9)). This is why an evacuation is useful in the case of a vacuum flask. Here, the vacuum is high enough so that a decrease in pressure causes a smaller thermal conductivity which, of course, has to be minimized to reach the best thermal isolation possible.

3.4 Thermal conductivity of copper, water, air and concrete

We are going to take a look at the thermal conductivity κ of these materials/substances. The values for κ are calculated using the formula:

$\kappa = \lambda_H / (c*\rho)$ (λ_H: heat conductance; c: specific heat capacity; ρ: mass density);

the values for λ_H, c and ρ are taken from Stöcker, Taschenbuch der Physik, pp. 216-222/724-728 (the values for concrete were taken from the Internet (several sources) and Paul A. Tipler: Physik).

Material/ substance	**Density at 20°C [kg/dm³]**	**heat conductance λ_H [W/(K*m)]**	**specific heat capacity c [J/(K*g)]**	**Thermal conductivity [m²/s]**
Copper	8.954	4.01	0.385	1.16
Water	1.003	0.60	4.187	0.14
Air	1.2928	0.02454	1.005	0.019
Concrete	≈ 2,5	≈ 0.5	≈0.9	0.22

Here are some examples for practical consequences:

- The thermal conductivity of copper is very high, hence it is used to produce heat sinks for example.
- The thermal conductivity of water is still relatively high, this makes water-cooling systems possible.
- Concrete has, unfortunately, a thermal conductivity within the same order of magnitude as the one of water. It is not a very well insulating building material.
- Air doesn't conduct heat very well. This means, that it can be used as an thermal insulator; the insulating effect of down jackets is based on this, for example.

3.5 Pascal and the pressure measurement on top of the Puy de Dôme (1463m)

Pascal's brother-in-law lived in Clermont-Ferrand, which is about 400m above sea level. Maybe Pascal estimated that the height difference to the top of the mountain would be about 1000m. His brother-in-law noticed that – after he had reached the top – the mercurial column inside his manometer had got about 10cm (maybe +/- 0.5cm) lower. Pascal could therefore have concluded, that the weight of a 10cm (+/- 0.5cm) mercurial column equals the weight of a 100000cm (+/- 50000cm) air column and that mercury is therefore 10000 (+/- c. 5500) times heavier than air. Therefore he could (after having obtained either the density of air or the one of mercury from some other experiment) have calculated the density of mercury respectively of air.

3.6 Range of molecular flow

The range of molecular flow is reached if the mean free path of a gas (under low pressure), which is calculated using the kinetic theory of gases, reaches the dimensions of the vessel the gas is inside. The equations derived from the kinetic theory of gases have to be modified in this case because collisions between two gas particles become negligible; the particles only interact with the vessel borders.

3.7 Evacuation of a vessel

The brass vessel of our experiment (volume V=3l) shall be evacuated as it was during our measurements for the effective pumping speed but now using a capillary of 1mm diameter. What pressure do we have to expect after 10 minutes of pumping in this case?
First of all we assume that we have viscous gas flow at least during the greater part of the measurement. As we have already described above, the diameter of the capillary then enters the conductance as fourth power, i.e. the conductance of the 1mm-capillary is 16 times smaller than the one for 2mm diameter.
Secondly we have a look at the effective pumping speed

$$S_{eff} = \frac{1}{\frac{1}{S} + \frac{1}{L_{tube}} + \frac{1}{L_{capillary}}} \stackrel{L_{capillary} << L_{tube}, S}{\approx} L_{capillary} .$$

Using this approximation we find that the effective pumping speed with the 1mm-capillary is 16 times lower than the one with the 2mm-capillary. Since the pressure follows the equation

$$p(t) = p_0 \cdot \exp\left(-\frac{S_{eff}}{V} \cdot t \right)$$

it is quite easy to estimate the pressure with the 1mm-capillary by using the measured curve for the 2mm-capillary:

$$p_{1mm}(10\min) = p_{2mm}\left(\frac{10\min}{16} \right) = p_{2mm}(37.5\text{s})$$

Thus, the only thing we have to do is to interpolate our curve for the 2mm-capillary. For this (rather rough) estimation we do not calculate a regression function once more but we determine the pressure graphically from the diagram:

$$p_{2mm}(37.5s) \approx 10\text{hPa} .$$

This is the pressure we expect after 10 minutes of pumping using the 1mm-capillary.

3.8 Mean free path λ at $4*10^{-11}$hPa

The mean free path equals (according to eq. (6) of the instructions):

$$\lambda = \frac{1}{\sqrt{32}\rho F}$$

where ρ is the particle number density (in 1/m³) and F the cross-section of a molecule.

Given an approximate value for λ of 10^{-4} cm (=: λ_1) at a pressure of p_1 = 1hPa (see instructions, p.4, section “Medium vacuum”), we can calculate the mean free path λ at $p = 4*10^{-11}$hPa:

$$\frac{\lambda}{\lambda_1} = \frac{\rho_1}{\rho} = \frac{p_1}{p} \Rightarrow \lambda = \frac{p_1}{p} \cdot \lambda_1 = 2.5 \cdot 10^6\,\text{m}$$

4. Appendix A: Explanation of the macro

Firstly, we take a look at the macro source code and the corresponding spreadsheet, which contains:

- The measured data
- A button “Calculate p!” for starting the macro
- Two empty table columns (H17-Hxx, I17-Ixx) for entering *I* values and showing the calculated *p* values. Three calculated pairs of variates are already shown.

Note that the red measured values aren’t taken into account, as already said before.

p [mbar]	I [mA]	lg(p/mbar)	lg(I/mA)	ΔI [mA]	Δp [mbar]
1,00E-04	2,7	-4,00	0,43	-	-
5,00E-03	3,2	-2,30	0,50	0,7	0,002
0,020	3,9	-1,70	0,59	0,7	0,005
0,031	4,4	-1,51	0,64	0,7	0,005
0,040	4,9	-1,40	0,69	0,7	0,005
0,070	5,7	-1,15	0,76	0,7	0,005
0,080	6,0	-1,10	0,78	0,7	0,005
0,10	6,6	-1,00	0,82	0,7	0,02
0,15	7,8	-0,82	0,89	0,7	0,02
0,23	9,6	-0,64	0,98	0,7	0,02
0,32	11,2	-0,49	1,05	0,7	0,02
0,55	14	-0,26	1,15	3,3	0,02
0,92	17	-0,04	1,23	3,3	0,02
1,06	19	0,03	1,28	3,3	0,04
1,54	22	0,19	1,35	3,3	0,04
8	33	0,90	1,52	3,3	4
16	37	1,19	1,57	3,3	4
28	43	1,44	1,63	3,3	4
37	45	1,57	1,65	3,3	4
59	48	1,77	1,68	3,3	4
81	50	1,91	1,69	3,3	4
101	51	2,00	1,70	3,3	4

Calculate p!

enter I values:	> p values (calculated):
4	0,016275171
5	0,044216977
34	9,469177399

p [mbar]	P [mV]	ΔP [mV] (err. propag.)	Δp [mbar]
1,00E-04	0,2	-	-
5,00E-03	0,3	0,13	0,002
0,020	0,5	0,16	0,005
0,031	0,6	0,18	0,005
0,040	0,7	0,21	0,005
0,070	1,0	0,24	0,005
0,080	1,1	0,25	0,005
0,10	1,3	0,28	0,02
0,15	1,8	0,33	0,02
0,23	2,8	0,40	0,02
0,32	3,8	0,47	0,02
0,55	6	2,79	0,02
0,92	9	3,35	0,02
1,06	11	3,76	0,04
1,54	15	4,40	0,04
8	33	6,53	4
16	41	7,35	4
28	54	8,42	4
37	61	8,91	4
59	69	9,50	4
81	74	9,80	4
101	77	10,00	4

The source code consists of three procedures (respectively functions). The first two ones return the slope value *m* and the axis intercept value *t* of a straight line which runs through two points (x_1,y_1); (x_2,y_2).

```
Static Function calcm(x1, y1, x2, y2)
    calcm = (y2 - y1) / (x2 - x1)
End Function

Static Function calct(x1, y1, x2, y2)
    calct = y1 - calcm(x1, y1, x2, y2) * x1
End Function
```

The main code of the macro is included in the event handler for a click onto the button included in the spreadsheet:

```
Private Sub CommandButton1_Click()

j = 17

While (Not ((IsEmpty(Range("H" + Format(j)).Value)) Or (Range("H" + Format(j)).Value = "")))
```

The first thing we introduce into the code is a counter j, which is the row index of the cells H17-Hxx, I17-Ixx; the program is repeated for every *I* value found in H17-Hxx (while…wend). (The format function converts integers to strings needed in the argument for Range().)

Now, the program reads out the *I* value and decides which range it is in and what it should do (see 2.1).

```
z = Range("H" + Format(j)).Value

If z > 3.55 Then
If z < 25 Then
```

This is the case in which the “xmgrace curve” can be applied…

```
  Range("I" + Format(j)).Select
  ActiveCell.FormulaR1C1 = ((z ^ 2) - 10.7578) / 322.098

End If
End If

If z <= 3.55 Then
```

In this case, we always have to take a straight line through the two measured points with smallest *I* values, because these are the only two points available for an interpolation in this range for *I*.

The slope of the straight line and its axis intercept are calculated out of the values ln(*p*) / ln(*I*); the obtained “function” *p*(*I*) is calculated according to eq. (III) (paragraph 2.1) of this document, and ActiveCell.FormulaR1C1 is assigned the result.

```
  Range("I" + Format(j)).Select
  ActiveCell.FormulaR1C1 = (z ^ calcm(Range("D3").Value, Range("C3").Value,
  Range("D4").Value, Range("C4").Value)) * (10 ^ calct(Range("D3").Value,
  Range("C3").Value, Range("D4").Value, Range("C4").Value))

End If

If z >= 25 Then
```

The calculation in this case uses the same system of linear interpolation in an imaginary logarithmic diagram. However, the two measured points to be used have to be searched for in B23…B16.

```
  a = "22"
  b = "23"

  For counter = 22 To 16 Step -1

    If (Range("B" + Format(counter)).Value <= z) And (Range("B" +
        Format(counter + 1)).Value > z) Then
      a = Format(counter)
      b = Format(counter + 1)
    End If

  Next counter
```

As one can see, if the current value is above all measured values, the highest measured values will be used for the interpolation, as described under (2.1).

Now we still have to do the calculation as above…

```
  Range("I" + Format(j)).Select
  ActiveCell.FormulaR1C1 = (z ^ calcm(Range("D" + a).Value,
    Range("C" + a).Value, Range("D" + b).Value, Range("C" + b).Value)) *
    (10 ^ calct(Range("D" + a).Value, Range("C" + a).Value,
    Range("D" + b).Value, Range("C" + b).Value))

End If

j = j + 1
Wend

End Sub
```

…and we're finished.